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RESONANT TUNNELING OF ULTRASOUND IN THREE-DIMENSIONAL PHONONIC CRYSTALS

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Recently, there has been a growing interest in studying phononic crystals, composite materials possessing periodic structure on length scales comparable with sonic or ultrasonic wavelengths[1]. In analogy with atomic and photonic crystals, the periodicity of the structure gives rise to phononic band gaps - ranges of frequencies in which acoustic or elastic wave propagation is forbidden. As was shown by S. Yang *et al.*[2], at the band gap frequencies there is still some signal traveling through a phononic crystal providing the thickness is not too great. The above authors also convincingly demonstrated that in the band gap acoustic waves are transmitted by tunneling, an effect that is analogous to tunneling of particles through a barrier in Quantum Mechanics. Another striking quantum mechanical phenomenon is the so-called resonant tunneling, which occurs when a particle of energy E is incident on a double barrier of height V, with $E < V$. For certain values of particle energy the transmission coefficient exhibits very sharp resonant peaks for which the transmission probability reaches unity. Can a similar phenomenon be observed for classical ultrasound waves? One can imagine an arrangement of two phononic crystals separated by a continuous media, so that at frequencies corresponding to the band gap such an arrangement would mimic the double barrier of Quantum Mechanics. Here we present the experimental study of the transport of the ultrasonic waves through such a "double" three-dimensional (3D) phononic crystal using a pulsed ultrasound technique.

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EXPERIMENTS

Our phononic crystals are made of 0.8-mm-diameter tungsten carbide beads immersed in water. Due to the big difference in both sound velocity and density of the constituent materials, a high scattering contrast is ensured. All crystals had a face-centered-cubic (fcc) structure, with layers of beads arranged in such a way that sound was incident along a body diagonal, or [111] direction. The sample was placed horizontally on top of a plastic substrate, thick enough to ensure the time separation of signals through sample and multiple reflections in the substrate. The sample itself consisted of two phononic crystals with a 7.05-mm-thick aluminum spacer sandwiched in between. This arrangement was then placed in a large water tank between two ultrasonic transducers.

RESULTS

The transmission coefficient as a function of frequency was determined from the ratio of the amplitudes of Fourier transforms of the input and transmitted pulses, and was then compared with the theoretical predictions of Multiple Scattering Theory (MST)[3]. MST is based on calculating the scattered waves from a representative scatterer (a sphere in our case) of the total incident wave field, including both the externally incident wave and the waves incident from all other scatterers in the crystal. Since the scattering of acoustic waves by an elastic sphere is an exactly solvable problem, MST allows one to reliably calculate the band structure of our phononic crystals as well as the transmission through the crystals. Theoretical calculations based on MST predict the existence of a band gap along the [111] direction between 0.8 and 1.2 MHz, in good agreement with experiment[2]. The transmission coefficient through the double crystal is presented over this frequency range in Fig. 1. The data curve clearly shows the large dip in transmission corresponding to the band gap, as well as a resonant peak in the middle of the band gap. The position of the peak corresponds within a few percent to the frequency at which the ultrasonic wavelength is equal to the thickness of the aluminum spacer. The slight difference in the resonant frequencies can be attributed to the phase shift acquired by the waves upon reflection from the interfaces between aluminum and each crystal. The fact that the magnitude of the resonant peak is not unity can be explained by absorption in our phononic crystals, which will be discussed in more detail below.

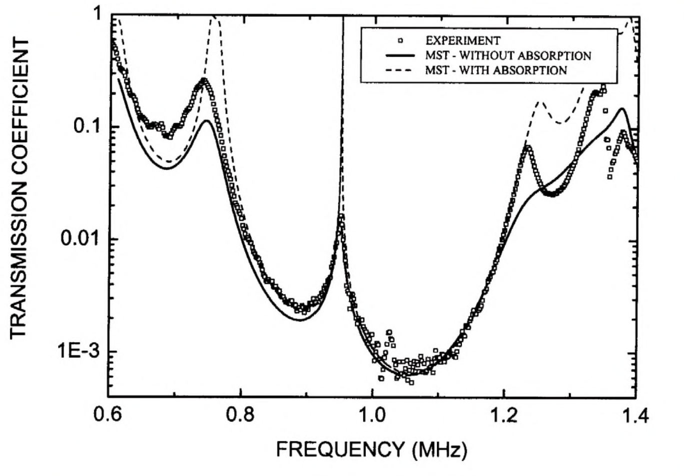

Fig. 1 Transmission coefficient through the double phononic crystal. Data (symbols) are compared with the theoretical predictions of MST without (dashed line) and with (solid line) absorption.

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Since we are using a pulsed technique, we can also investigate the dynamics of resonant tunneling by measuring the group time and group velocity as a function of frequency. To this end, the pulses were digitally filtered by a narrow (around 0.01 MHz) Gaussian bandwidth. The group time was then found by measuring the time delay between the Gaussian sample and input pulses, from which we then calculated the group velocity. As can be seen from Fig. 2(a), the group time exhibits sharp peak at the resonant frequency, implying that on-resonance the wave pulses are significantly slowed down, whereas off-resonance they travel very fast. The very fast group velocity off-resonance is an indication that tunneling is involved.

As was mentioned above, the magnitude of the resonant peak is much less than unity due to absorption in crystals. This conclusion is supported by the results of theoretical calculations based on MST. If no absorption were present in the crystal, magnitude of the resonant peak is predicted to be unity, as shown by the dashed line in Fig. 1. Introducing absorption inside crystals reduces magnitude of the peak. This case is represented by the solid curve in Fig. 1. Magnitude of absorption is consistent with the observed group velocity through a single phononic crystal[2]. Good agreement between theory and experiment is observed. MST also allows us to calculate group time and group velocity from the derivative of the phase of the amplitude transmission coefficient with frequency. The solid curves in Fig. 2(a) and Fig. 2(b) represent theoretical predictions for group time and group velocity respectively; good agreement with the data is again observed.

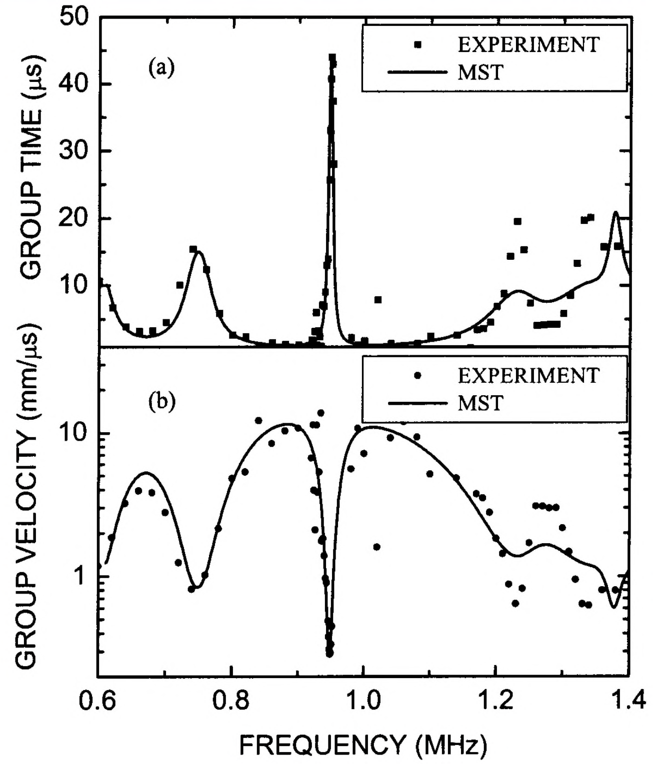

Fig. 2 **(a) Comparison of the measured group time (symbols) and theoretical prediction of MST (solid line),.(b) Comparison of the group velocity obtained from experiment (symbols) and calculated from MST (solid line).**

The main effect of absorption in the band gap of a phononic crystal is to substantially attenuate long multiple scattering paths, which makes the destructive interference of Bragg-scattered waves incomplete[2]. As a result, the main effect of absorption is to introduce a small amplitude propagating component in addition to the dominant evanescent mode that is responsible for the tunneling. In reference[2], it was shown that this two modes model was able to quantitatively predict the observed magnitude of the group velocity in a single phononic crystal at gap frequencies, by modeling wave transport as a parallel combination of a dominant tunneling process accompanied by a small propagating mode. In our resonant tunneling experiments, the effect of absorption in the phononic crystals is to lower the quality factor Q of the resonance, as absorption introduces leakage from the resonant cavity between the crystals via the propagating mode. Hence the resonant peak is broadened, and its maximum reduced. The effect of absorption on the group time can be similarly interpreted. In the absence of absorption, the group time at the resonant frequency (which is essentially the dwell time of a pulse in a resonant cavity) is predicted to increase exponentially, both in the quantum mechanical and acoustic cases, as thickness of the potential barrier or phononic crystal grows. By contrast, absorption cuts off this exponential increase because of the leakage via the propagating mode, and the group time increases less rapidly with thickness. To verify this effect of absorption on the group time, we ran a series of experiments on double crystals having 2, 3 and 4 layers in each of the constituent crystals. The data are in agreement with the theoretical prediction, giving additional evidence of the large but also subtle effects of absorption on resonant tunneling in acoustic systems.

CONCLUSIONS

In this paper our results on resonant tunneling in three-dimensional phononic double crystals are presented. A resonant peak is successfully observed. The position of the peak is in agreement with the general resonant condition that an integer number of half-wavelengths be approximately equal the thickness of the cavity between crystals. The theoretical calculations based on MST allow us to conclude that resonant peak magnitude is much less than unity due to absorption inside crystal, and a simple physical interpretation of the effects of absorption is presented.

ACKNOWLEDGEMENTS

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REFERENCES

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